Titles / Abstracts for Talks at TGDA@OSU 2016

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- <u>Speaker: Peter Bubenik</u>

Title: Higher Interpolation and Extension for Persistence Modules

Abstract: Persistence modules are the central algebraic object in topological data analysis. This motivates the study of the geometry of the space of persistence modules. We isolate an elegant coherence condition that guarantees the interpolation and extension of sets of persistence modules. This "higher interpolation" is a consequence of the existence of certain universal constructions. As an application, it allows one to compare Vietoris-Rips and Cech complexes built within the space of persistence modules. This is joint work with Vin de Silva and Vidit Nanda.

- <u>Speaker: Dan Burghelea</u>

Title: From Computer and Data back to Topology and Geometry; an alternative approach to Morse-Novikov theory and monodromy

Abstract: It is well known how basic Algebraic Topology and geometrization of Large Data led to Persistence Theory, a useful tool in Data Analysis.

In this talk I will explore the other direction; how Persistence Theory suggests and motivates refinements of some basic topological invariants, like homology and Betti numbers, and suggests alternative descriptions of others invariants, like monodromy, of mathematical relevance and with computational implications. The mathematics described is a part of what I refer to as an ALTERNATIVE to MORSE-NOVIKOV theory.

The refinements proposed are in terms of configurations of vector spaces for the relevant homologies, and in terms of polynomials for Betti numbers. The alternative description of monodromy is computer friendly, hence without the need of infinite objects (infinite cyclic cover). A few applications of these refinements in topology, geometric analysis and dynamics might be indicated.

- <u>Speaker: Gunnar Carlsson</u>

Title: Excision and product results for persistent homology

Abstract: Motivated by work on data sets coming from the biology and evolution, we have some results concerning the persistent homology of trees and also products of finite metric spaces.

- Speaker: Justin Curry

Title: Realization Problems in Persistence

Abstract: In this talk I will introduce the realization problem in persistence, which asks what isomorphism classes of diagrams of vector spaces can be realized by diagrams of topological spaces and continuous maps. In particular, one could ask what barcodes can be obtained by filtering a simplicial complex or by studying the level-set persistence of a map. I will review existing results for point clouds, and present some results of my own, obtained in collaboration with Ulrich Bauer and Hans Reiss. These results include studying the level-set barcodes realized by a stratified space and real-valued map, the space of barcodes realized by filtering a manifold by Gauss curvature, and the space of barcodes realized by Morse functions. To address which barcodes can be realized as the level-set barcodes of a Morse function f, I will present two constructions. One construction uses handlebody theory. The other construction is more novel and uses a cosheaf of spaces over the Reeb graph of f, which incidentally makes headway into a problem posed by Arnold. Additionally, this construction offers a vision for extending Mapper degree by degree (in analogy with Postnikov towers), offering a potentially powerful new tool in topological data analysis.

- Speaker: Carina Curto

Title: Emergent dynamics from network connectivity: a minimal model

Abstract: Many networks in the brain display internally-generated patterns of activity -- that is, they exhibit emergent dynamics that are shaped by intrinsic properties of the network rather than inherited from an external input. While a common feature of these networks is an abundance of inhibition, the role of network connectivity in pattern generation remains unclear.

In this talk I will introduce Combinatorial Threshold-Linear Networks (CTLNs), which are simple "toy models" of recurrent networks consisting of threshold-linear neurons with binary inhibitory interactions. The dynamics of CTLNs are controlled solely by the structure of an underlying directed graph. By varying the graph, we observe a rich variety of emergent patterns including: multistability, neuronal sequences, and complex rhythms. These patterns are reminiscent of population activity in cortex, hippocampus, and central pattern generators for locomotion. I will present some theorems about CTLNs, and explain how they allow us to predict features of the dynamics by examining properties of the underlying graph. Finally, I'll show examples illustrating how these mathematical results guide us to engineer complex networks with prescribed dynamic patterns.

- Speaker: Vin de Silva

Title: Reeb cosheaves

Abstract: I shall talk about Reeb graphs, and the benefits of regarding them as set-valued cosheaves on the real line. For instance, there is a smoothing operation and an interleaving distance for these cosheaves, which can be interpreted as a smoothing operation and an interleaving distance for Reeb graphs. This is joint work with Elizabeth Munch and Amit Patel. If time allows, I will discuss various extensions of these ideas, worked out with the additional collaboration of Anastasios Stefanou; and also of Pomona College undergraduates Song Yu and Dmitriy Smirnov.

- Speaker. Herbert Edelsbrunner

Title: Expected sizes of Poisson--Delaunay mosaics and their discrete Morse functions

Abstract. Mapping every simplex in the Delaunay mosaic of a discrete point set to the radius of the smallest empty circumsphere gives a generalized discrete Morse function. Choosing the points from a Poisson point process in R^n, we study the expected number of simplices in the Delaunay mosaic as well as the expected number of critical simplices and non-singular intervals in the corresponding generalized discrete gradient. Observing connections with other probabilistic models, we study general properties of the discrete gradient and obtain precise expressions for the expected numbers in low dimensions. In particular, we get the expected numbers of simplices in the Poisson--Delaunay mosaic in dimension n <= 4.

(Joint work with Anton Nikitenko and Matthias Reitzner.)

- Speaker: Jeff Erickson

Title: Untangling Planar Curves

Abstract: Any generic closed curve in the plane can be transformed into a simple closed curve by a finite sequence of local transformations called homotopy moves. We prove that simplifying a planar curve with n self-crossings requires Theta(n^{3/2}) homotopy moves in the worst case. The best bounds previously known were a 100-year-old O(n^2) upper bound due to Steinitz and the trivial Omega(n) lower bound. Our lower bound also implies that Omega(n^{3/2}) degree-1 reductions, series-parallel reductions, and Delta-Y transformations are required to reduce any planar graph with treewidth Omega(sqrt{n}) to a single edge, matching known upper bounds for rectangular and cylindrical grid graphs. Finally, we prove that Omega(n^2) homotopy moves are required in the worst case to transform one non-contractible closed curve on the torus to another; this lower bound is tight if the curve is homotopic to an embedding.

This is joint work with Hsien-Chih Chang, to be published at SOCG 2016.

- Speaker: Jennifer Gamble

Title: TDA in healthcare

Abstract: This talk will give some examples of applications of topological data analysis to the field of healthcare. In addition to the foundational 'TDA' algorithms of persistent homology and mapper, this includes TDA in the broader sense: explicitly taking a topological/geometric approach to a data analysis problem. Examples will include understanding and managing clinical variation in a hospital system, and describing complicated patterns of denials of medical claims. General approaches, including the incorporation of alternative metrics or application-relevant filtrations or mapper functions will also be discussed.

- Speaker: Robert Ghrist

Title: Sheaf Theory: Applications & Computations

Abstract: Applied & computational topology has done a good job at demonstrating that computing the homology of a complex (or sequence of complexes) is useful and computationally feasible. What comes next? This talk will argue that the next set of tools for applied topology will come from sheaf theory. The talk will follow along two lines: first, that sheaves, and, in particular, sheaf cohomology, is useful in several applications; and second, that there are good algorithms for computing sheaf cohomology (that in fact impact how one computes, inter alia, persistent homology).

- Speaker: Sanjeevi Krishnan

Title: Higher Group Completions of Monoids

Abstract: Every connected, finite CW complex is homotopy equivalent to the classifying space of a monoid [McDuff, 1979], a set with an associative and unital multiplication. Thus group completions of monoids correspond to fundamental groups of based spaces. We discuss concrete algorithms for computing (co)homology and higher homotopy as algebraic constructions on monoids.

- Speaker: Michael Lesnick

Title: Algebraic Stability of Zigzag Persistence Modules

The stability theorem for persistent homology is a central result in topological data analysis. While the original formulation of the result concerns the persistent homology of \mathbb{R}-valued functions, the result was later cast in a more general algebraic form, in the language of persistence modules and interleavings. In this work, we establish an analogue of this algebraic stability theorem for zigzag persistence modules. To do so, we functorially extend each zigzag persistence module to a two-dimensional persistence module, and establish an algebraic stability theorem for these extensions. As an application of our main theorem, we strengthen a result of Bauer, Munch, and Wang on the stability of the persistent homology of Reeb graphs. Our main result also yields an alternative proof of the stability theorem for level set persistent homology of Carlsson et al.
This is joint work with Magnus Botnan.

- Speaker: Sayan Mukherjee

Title: Learning Group Actions and the Geometry/Cohomology of Synchronization Problems

Abstract: We present a unified geometric treatment of a large class of synchronisation problems, which we support further with cohomological interpretations. We show that the existence of solutions to the synchronisation problem can be described very naturally in terms of group-valued holonomy of loops on the vertex set, which suggests a very interesting new geometric interpretation of the problem. We present an interpretation of the problem as a question of existence of compatible global sections of a fibration constructed from this data. We demonstrate that our fibration can be regarded as a simplicial shadow of a fibre bundle. Motivated by differential geometry, we use this data to construct a twisted de Rham type complex and and associated bundle-valued twisted Laplace operator. We prove that the space of solutions of the synchronisation problem is the same as the space of twisted harmonic forms in our setting. Furthermore, we formulate a new class of statistical inference problems closely related to synchronization, that allow for the inference of group actions from observations; we also propose a message-passing algorithm to tackle such problems and demonstrate its applicability in practice.

This is joint work with Jacek Brodzki and Tingran Gao

- Speaker: Elizabeth Munch

Title: Applications of Persistence to Time Series Analysis

Abstract:
A time series is simply a stream of data. However, the type of output of this stream can be many different things. The most commonly studied and most understood form of time series is real-valued, however this is certainly not the only way that time series are presented in practice. For instance, a movie can be interpreted as a matrix-valued time series.

In this talk, we will look at two applications in different domains. Both of these applications utilize persistent homology to provide analysis even though the types of time series under study are quite different. First, we will discuss the phenomenon of chatter in machining dynamics. Chatter is the undesirable behavior exhibited by a cutting tool which is characterized by large amplitude vibrations that result in non-smooth metal parts, as well as an intense noise. Combining Taken's embedding theorem, persistence, and machine learning methods gives 97% accuracy when attempting to predict and prevent this behavior. Second, we will use persistence to quantify a diurnal cycle recently observed in IR hurricane data. In this case, we turn a matrix valued time series into a persistence diagram valued time series and investigate the resulting periodic behavior in persistence diagram space.

- Speaker: Vidit Nanda

Title: The Discrete Flow Category

Abstract: Large-scale homology computations are often rendered tractable by discrete Morse theory. Every discrete Morse function on a given cell complex X produces a Morse chain

complex whose chain groups are spanned by critical cells and whose homology is isomorphic to that of X. However, the space-level information is typically lost because very little is known about how critical cells are attached to each other. In this talk, we discretize a beautiful construction of Cohen, Jones and Segal in order to completely recover the homotopy type of X from an overlaid discrete Morse function.

- Speaker: Amit Patel

Title: Semicontinuity of Persistence Diagrams

Abstract: The persistence diagram is very different in philosophy from the barcode. Suppose we have a constructible persistence module of vector spaces. Its barcode is its list of indecomposables. Its persistence diagram is an encoding of all persistent vector spaces. In the setting of vector spaces, we know that these two notions are equivalent. However, we quickly run into problems if we try to generalize the barcode beyond the setting of vector spaces. In this talk, I will generalize the persistence diagram to the setting of constructible persistence modules valued in any symmetric monoidal category. For example, the category of sets, the category of vector spaces, and the category of abelian groups are symmetric monoidal categories. As an immediate consequence, we can finally talk about persistent homology over integer coefficients!

- Speaker: Jose Perea

Title: Sparse Cech filtrations, persistent cohomology and projective coordinates

Abstract:
One of the main challenges in topological data analysis is to turn computed topological features, such as barcodes, into insights about the data set under analysis.
We will show in this talk how the persistent cohomology of sparse Cech filtrations (introduced recently by D. Sheehy et. al.), in dimensions 1 and 2, can be used to construct robust representations of the data in the real and complex projective spaces. Examples will be presented in order to illustrate how projective coordinates provides a framework for topology-driven nonlinear dimensionality reduction, and geometric model generation.
This work extends results of V. de Silva, D. Morozov and M. Vejdemo-Johansson on persistent cohomology and circular coordinates.

- Speaker: Benjamin Schweinhart

Title: Topological Similarity of Random Cell Complexes, and Applications

Abstract: Although random cell complexes occur throughout the physical sciences, there does not appear to be a standard way to quantify their statistical similarities and differences. I'll introduce the method of swatches, which describes the local topology of a cell complex in terms of probability distributions of local configurations. It allows a distance to be defined which measures the similarity of the local topology of cell complexes. Convergence in this distance is related to the notion of a Benjamini Schramm graph limit. In my talk, I will use this to state universality conjectures about the long-term behavior of graphs evolving under curvature flow, and to test these conjectures computationally. This system is of both mathematical and physical interest.

If time permits, I will discuss other applications of computationally topology to curvature flow on graphs, and describe recent work on a new notion of geometric graph limit.

- Speaker: Don Sheehy

Title: Hierarchical Sampling Strategies

Abstract:
In this talk I will discuss techniques and heuristics for subsampling metric data as well as a space of tree-like data structures that one might build on top of such samples, generalizing cover trees, net trees, navigating nets, deformable spanners, and some classes of hierarchical spanners.

This is joint work with Reza Jahanseir

- Speaker: Bei Wang

Title: Categorical Representations of Reeb Space and Mapper: Convergence and Multivariate Data Analysis

Abstract:
The Reeb space, which generalizes the notion of a Reeb graph, is one of the few tools in topological data analysis and visualization suitable for the study of multivariate scientific datasets. First introduced by Edelsbrunner et al., it compresses the components of the level sets of a multivariate mapping and obtains a summary representation of their relationships. A related construction called mapper (Singh et al.), and a special case of the mapper construction called the Joint Contour Net (Carr et al.) have been shown to be effective in visual analytics. Mapper and JCN are intuitively regarded as discrete approximations of the Reeb space, however without formal proofs or approximation guarantees. An open question has been proposed by Dey et al. as to whether the mapper construction converges to the Reeb space in the limit.

We are interested in developing the theoretical understanding of the relationship between the Reeb space and its discrete approximations to support its use in practical data analysis. Using tools from category theory, we formally prove the convergence between the Reeb space and mapper in terms of an interleaving distance between their categorical representations. Given a sequence of refined discretizations, we prove that these approximations converge to the Reeb space in the interleaving distance; this also helps to quantify the approximation quality of the discretization at a fixed resolution.